Power Balance ICP Model

Dr. Bhupal Singh[1], Dr. K.B. Sahay[2] and **Dr. Sneh Anand[3]**
[1]Department of Electrical and Electronics Engineering, Ajay Kumar Garg Engineering College,
PO Adhyatmik Nagar, Ghaziabad 201009 UP India
[2, 3] Indian Institute of Technology, New Delhi 110016 India
[1]singhbhupal@akgec.ac.in, [2]kbsahay@cbme.iitd.ac.in, [3]sneha@cbme.iitd.ac.in

Abstract -- **In hydrocephalus and severe brain injuries, Intracranial Pressure (ICP) monitoring is commonly used for neurological care, as raised ICP affects cerebral perfusion besides compressing brain tissues. There is a compelling demand of the biomechanical parameters estimation for clinical use and their impact on ICP. Several models have been reported to simulate certain aspects of ICP. However, there are very few models that encompass so many parameters and still remain simple. A versatile system modeling technique of power balance has been employed to develop a biomechanical model of ICP for its possible application in the clinical parameter estimation. The model has been validated employing reported data.**

Keywords: Intracranial pressure, Cerebro-spinal fluid, Power balance, elastance coefficient

I. INTRODUCTION

IN hydrocephalus and severe brain injuries ICP monitoring is commonly used for neurological care (Becker 1977, Miller 1987), as raised ICP affects cerebral perfusion (Ursino, et al. 1997) and also compresses brain tissues (Hakim *et al.* 1984). The prevailing views regarding the genesis of ICP can be found in the literature (Agarwal *et al.* 1969, Berman *et al.* 1984; Guinane 1972, Czosnyyka 2000; Ursino 1988; Ursino *et al.* 1997). The present study proposes a more realistic and yet tractable model involving important clinical parameters and correlates of the ICP. The present model developed applying fluid power balance to CSF system also incorporates additional features like visco-elasticity of the brain tissues.

II. MODEL FORMULATION

Cranio-spinal system, the contents of cranial and spinal cavity is divided into dual flow system namely: Blood flow and CSF flow systems. The later is of the primary interest for present study. CSF system is in the state of power balance (exchange) with: (i) cerebral blood volume perturbation through cerebral vascular system from within, (ii) spinal epidural volume perturbation via dural membrane exteriorly and (iii) CSF volume accumulation itself. Therefore, capturing minute details of ICP's fluid dynamic behavior, fluid power balance is thought to be most appropriate one along with certain simplifying but valid assumptions.

A. Power Balance Model Structure

Let at any instant *t*, $\overline{P_f}$ and P_f be respectively instantaneous spatial mean ICP and lateral ventricular ICP in supine position of the adult human. Let these pressures be related through a spatial pressure distribution factor, k_f as follows

$$\overline{P_f} = k_f P_f \tag{1}$$

The pressure $\overline{P_f}$ resists to the total volume changes namely: cerebral blood volume $(\frac{dv_b}{dt})$, CSF volume $(\frac{dv_f}{dt})$ within crano-spinal-space and additionally, subdural blood volume perturbations $(\frac{dv_e}{dt})$.

Total fluid power available $\overline{P_f}(\frac{dv_b}{dt} + \frac{dv_f}{dt} + \frac{dv_e}{dt})$ to deform the CNS tissue complex is utilized to: (i) deform CNS tissues and dural membrane and (ii) meet-out viscoelastic power losses of brain tissue complex.

However, rate of change of deformational potential energy $(\frac{du}{dt})$ of the CNS tissue complex and its viscoelastic power losses (Γ_f), fluid power balance gives

$$\overline{P_f}(\frac{dv_b}{dt} + \frac{dv_f}{dt} + \frac{dv_f}{dt}) = \frac{du}{dt} + \Gamma_f \tag{2}$$

B. Elastance Coefficient and Its Significance

Equation (2) can be simplified if $\overline{P_f}$ is assumed to be related as follows

$$\overline{P_f} = f(u) \Rightarrow \frac{d\overline{P_f}}{dt} = \frac{\partial \overline{P_f}}{\partial u}\frac{du}{dt} \Rightarrow \frac{d\overline{P_f}}{dt} = \lambda_f \frac{du}{dt} \tag{3}$$

where $\lambda_f = \frac{\partial \overline{P_f}}{\partial u}$, is a pressure energy gradient of the CNS tissue complex which is related to elastance coefficient as proposed by Avezaat *et al*. (1979).

C. Viscoelastic Aspects of Brain Tissue Complex

Assuming, the brain-tissue complex be pieces-wise linear-viscoelastic. The spatial mean of viscoelastic stress (σ_{vis}) of brain tissue complex and corresponding shear rate ($\frac{d\varepsilon}{dt}$)

are related as follows $\sigma_{vis} = \eta_{bt} \frac{d\varepsilon}{dt}$ (4)

where η_{bt}, is viscoelastic coefficient of brain tissue and ε is mean shear strain of the brain tissue complex.

Let viscoelastic energy loss for infinitesimal change in shear strain $d\varepsilon$ due to viscoelastic stress σ_{vis} of brain tissue with volume v_{bt} be *dw*.

Then, from the concept of elastic potential energy

$$dw = \sigma_{vis} v_{vt} d\varepsilon \Rightarrow \frac{dw}{dt} = \eta_{bt} v_{bt} (\frac{d\varepsilon}{dt})^2 \quad (5)$$

Further, mean shear strain ε is approximated by the relative CSF volume perturbation within the CSF System.

$$\varepsilon = \frac{v_f - v_{fn}}{v_{fn}} \quad (6)$$

where v_f and v_{fn} are respectively instantaneous and mean CSF volumes.

Therefore, equations (5) and (6) give

$$\frac{d\Gamma_f}{dt} = \frac{\eta_{bt} v_{bt}}{v_{fn}} (\frac{dv_f}{dt})^2 = B_f (\frac{dv_f}{dt})^2 \quad (7)$$

where $B_f = \frac{\eta_{bt} v_{bt}}{v_{fn}}$

D. CSF Balance

q_{fs} (*ml/s*) is the CSF secreted actively in the ventricles I_i, mock CSF being infused in lateral ventricle in supline position and q_{fr} (*ml/s*) is CSF drained into superior saggital sinuous with blood pressure P_{vs}, (*mmHg*) via arachnoid villi, and hydraulic resistance offered, R_{fr} (*mmHg.s/ml*) then,

$$q_{fr} = \frac{(P_f - P_{vs})}{R_{fr}}.$$

Thus, applying CSF fluid balance gives

$$\frac{dv_f}{dt} = (I_i + q_{fs} - \frac{(P_f - P_{vs})}{R_{fr}}) \quad (8)$$

But equations (1-3 & 7) give

$$\frac{dP_f}{dt} = \lambda_f [P_f (\frac{dv_b}{dt} + \frac{dv_f}{dt} + \frac{dv_e}{dt}) - \frac{B_f}{k_f} (I_i + q_{fs} - (P_f - P_{vs})/R_{fr})^2] \quad (9)$$

However, elastance of the CSF system E_f (*mmHg/ml*) is defined as

$$E_f = \frac{dP_f}{dv_f} \quad (10)$$

Therefore equations (9) and (10) give

$$E_f = \frac{dP_f}{dv_f} = \lambda_f P_f (1 + \frac{dv_b}{dv_f} + \frac{dv_e}{dv_f} + \frac{B_f'}{R_{fr}}) - \lambda_f B_f' (I_i + q_{fs} + P_{vs}/R_{fr}) \quad (11)$$

where $B_f' = \frac{B}{k_f}$

E. Predictions of Proposed Model

The equation (11) predicts four important things: 1) the elastance–pressure relationship is linear for given constant infusion I_i and other parameters involved, vary insignificantly.

2) Elastance-pressure line's $E_f = aP_f - b$ slope (a) and intercept (b) are negatively correlated for infusion tests with common factor of λ_f .

Where $a = \lambda_f (1 + \frac{dv_b}{dv_f} + \frac{dv_e}{dv_f} + \frac{B_f'}{R_{fr}})$ and $b = \lambda_f B_f' (I_i + q_{fs} + P_{vs}/R_{fr})$

A prolonged constant rate CSF infusion in animals produces plateau pressure wave, which attains constant steady state value of ICP.

The value of steady state plateau pressure P_{fv} is obtained by solving equation (11) for pressure:

$$P_{fv} = \frac{(E_f/\lambda_f) + B_f' (I_i + q_{fs} + P_{vs}/R_{fr})}{(1 + \frac{dv_b}{dv_f} + \frac{dv_e}{dv_f} + \frac{B_f'}{R_{fr}})} \quad (11)$$

This equation clearly indicates P_{fv} include both viscous as well elastic components smaller the value of I_i more dominant the later one will be.

4) P_{fv} is affected by viscoelastic behavior of the brain tissues and other parameters as well. Subsequent section validates the proposed model, employing reported human clinical data.

III. MODEL VALIDATION AND DISCUSSION

The power balance biomechanical model of ICP predicts that CSF elastance (E_f) in elastance-ICP plane can be approximated by line with a positive slope and negative intercept for a constant infusion rate, which has been reported in experimental studies (Avejaat *et al*. 1979). Present model also spells clearly that there is a common multiplying factor λ_f between the slope and the intercept with opposite sign.

A. Validation of Key Model Predicted Features

These predictive features of the proposed model will be authenticated employing clinical data in human reported by Paltsev *et al.* (1982). Paltsev *et al.* (1982) in their clinical study measured the ICP at the level of ventricle (left) in supine position of the brain tumors patients post operatively by ventricle fluid injection method.

The slopes and intercepts of elastance-ICP line reported by the same investigator for two groups (I & II) comprising of sixteen human of subjects each are analyzed for possible correlation using MATLAB software. A matrix 16x2 consisting of two columns of the slopes and the intercepts as variables with sixteen entries each are evaluated for possible correlation for every group separately. The MATLAB *xcorr(slope; intercepts)* returns correlation matrix (*R)* along with corresponding *p-value* matrix. If the p-value of corresponding entry of *R* matrix is smaller, correlation is considered to be stronger**.** The diagonal value gives the auto-correlation coefficient and whereas off-diagonal entry of *R* matrix gives cross-correlation between slope and intercept variables. The *R* and *p-value* matrices for two set of data as obtained are presented as follows.

Group -I

$$R=\begin{bmatrix} 1.000 & -0.4576 \\ -0.4576 & 1.000 \end{bmatrix} \text{ and } p-value=\begin{bmatrix} 1.0000 & 0.0746 \\ 0.0746 & 1.0000 \end{bmatrix}$$

In the *R* with diagonal values are autocorrelations of the slope and intercept entry with corresponding p-value being unity (high) implies that slopes and intercepts data are not correlated to them self. However, off-diagonal entries (-0.8271) shows cross correlation between slope and intercept variables corresponding p-value (0.0746) which is strong and negative.

Group -II

$$R=\begin{bmatrix} 1.000 & -0.4576 \\ -0.4576 & 1.000 \end{bmatrix} \text{ and } p-value=\begin{bmatrix} 1.0000 & 0.0746 \\ 0.0746 & 1.0000 \end{bmatrix}$$

Similarly for second group as well the correlation between slope and intercept is very strong and negative.

The (mean $\pm$ variance) values of slopes and intercepts for both the groups are respectively

Group –I Slope = 0.075 ± 0.011 and Intercept = 0.83 ± 0.12
Group –II Slope = 0.117 ± 0.018 and Intercept = 0.76 ± .05

The possible application of this model is in calculation of dynamic blood volume change by measuring the ICP alone, which is otherwise quite involved and tedious measurement.

However, its accuracy is limited by λ_f **.** If the CSF infusion rate is constant and continues for duration sufficiently larger then the cardiac synchronous blood volume changes and other artifacts are averaged out, the resultant ICP will be purely CSF accumulation. Hence, under such condition, it can be used to measure other parameters optimally by varying model parameters to best fit the measured ICP dynamically.

IV. CONCLUSION

The proposed model is validated through its critical predictions. These predictions are found in conformity to reported results. The plateau ICP wave's dependence on several parameters has been revealed by the present model which is also reported in experimental studies. This model integrated most of the significant features such as visco-elasticity without adding complexity to the model, which is a significant aspect. This model also gives the physical significance of the elastance-coefficient more precisely and above all the model can be employed for real time parameter estimation of elastance-coefficient CSF out flow resistance, and also used for blood volume perturbation measurement.

REFERENCES

[1] D.P. Becker *et al.*, "The outcome from severe head injury with early diagnosis and intensive management", *Journal of Neurosurgery,* vol.47, pp 491-502,1977.

[2] J.D.Miller, " IPC monitoring current status and future directions", Acta Neurochir (Wien), vol. 84, pp 80-85,1987.

[3] M . Ursino *et al.*,"A simple mathematical model of the interaction between intracranial pressure and cerebral hydrodymamics", *J. Appl.Physiol.*, vol.82, pp.1256-1269, 1997.

[4] S. Hakim *et al.*,"A biomechanical model of hydrocephalus and its relationship to treatment", In : K. Shapiro, T. Marmarou and H. Portnoy (eds), *Hydrocephalus*, Raven, New York , pp. 143-160, 1984.

[5] G. C. Agarwal, B. M. Berman and L. Stark, "A lumped parameter model of the cerebrospinal fluid system," *IEEE Trans. Biomed. Eng.*, vol.16, pp. 45–53, 1969.

[6] K. Ambarki, O. Baledent, G. Kongolo, R. Bouzerar, S. Fall and M.E. Meyer, "A new lumped-parameter model of cerebrospinal hydrodynamics during the cardiac cycle in healthy volunteers," *IEEE Trans. Biomed. Eng.,* vol, 54, pp. 483–491, 2007.

[7] O. Baledent, C. Gondry-Jouet, M.E. Meyer, G.D. Marco, D.L. Gars, M.C. Henry-Feugeas and I. Idy-Peretti, "Relationship between cerebrospinal fluid and blood dynamics in healthy volunteers and patients with communicating hydrocephalus," *Invest. Radiol.*, vol. 39, pp.45–55, 2004.

[8] C. J. J. Avezaat, J. H. M. van Eijndhoven and D.J. Wyper "Cerebrospinal fluid pulse pressure and intracranial volume

pressure relationship," *J. Neurol. Neurosurg. Pshychiatry*, vol. 42, pp, 687-700, 1979.

[9] M. Egnor, A. Rosiello and L. Zheng, "A model of intracranial pulsations," *Pediatr. Neurosurg*., vol. 35, pp. 284–298, 2001.

[10] I. Eugeny, C. Paltsevand. Biol.Sci and Edward B. Sirovsky, "Intracranial physiology and biomechanics- Clinical data on pressure-volume relationships and their interpretation," *Journal of Neurosurgery,* vol. 57, pp. 500-510 , 1982,

[11] J.E. Guinane, "An equivalent circuit analysis of cerebrospinal fluid hydrodynamics," *Amer. J. Physiol.*, vol. 223, pp. 425–430, 1972.

[12] J. Lofgren, C. V. Essen and N. N. Zwetnow, "Pressure volume curve of the CSF space in the dogs," *Acta Neurol. Scand.*, vol. 49, pp. 557-564, 1973.

[13] F. Loth, M.A. Yardimci and N. Alperin, "Hydrodynamic modeling of cerebrospinal fluid motion within the spinal cavity.," *J. Biomech. Eng.*, vol.123, pp.71–79, 2001.

[14] A. Marmarou, K. Shulman and R.M. Rosende, "A nonlinear analysis of the cerebral fluid system and intracranial pressure dynamics," *J. Neurosurg*., vol. 48, pp.332-344, 1978.

[15] A. Marmarou, K. Shulman and J. Lamorgese, "Compartmental analysis of compliance and outflow resistance of the cerebrospinal fluid system," *J. Neurosurg.*, vol. 43, pp. 523–533, 1975.

[16] A.L. Maset, A. Marmarou, J.D. Ward, S. Choi, H.A. Lutz, D.R. Brooks, J. Moulton, A. DeSalles, J.P. Muizelaar and H. Turner, "Pressure-volume index in head injury," *J. Neurosurg.*, vol. 67, pp.832–840,1987.

[17] R.W. Tain and N. Alperin, "Noninvarive intracranial compliance from MRI based measurements of transcranial blood and CSF flows: Indirect versus direct approach," *IEEE Trans. Biomed. Eng***,** vol.56, pp. 544-551, 2009

[18] M.E. Wagshul, J.J. Chen, M.R. Egnor, E.J. McCormack and P.E. Roche, "Amplitude and phase of cerebrospinal fluid pulsations: experimental studies and review of the literature," *J. Neurosurg.*, vol.104, pp. 810–819, 2006.

[19] A.T. Minassian, L. Dube, A.M. Guilleux, N. Whrmann, M. Ursion and L. Beydon L, "Changes in intracranial pressure and cerebral autoregulation in patients with severe traumatic brain injury," *Crit. Care Med*., vol. 30, pp. 1616-22, 2002.

[20] B.Berman *et al.*,"An integrated approach to intratranial hydraulic physiology", *Surg. Neurol.*,vol.22, pp. 83-95, 1984.

[21] M. Czosnyka, "Association between arterial and intracranial pressures", *British J. Neurosurgery,* vol.14, pp. 127-128, 2000.

[22] M. Ursino, "A mathematical study of human intracranial hydrodynamics Part I - the cerebrospinal fluid pulse pressure", *Ann. Biomed.Eng.,*vol.16, pp.379-401, 1988.

[23] M.Ursino *et al.*, "Intracanial pressure dynamics in patients with acute brain damage: A critical analysis with the aid of a mathematical model", *IEEE Trans Biomed Eng.,* vol.42 , pp. 529-540, 1995.

[24] H.G. Frieden *et al.*, "Volume pressure relationship of cerebrospinal space in human", *Neurosurgery,* vol.30, pp.351-366, 1983.

[25] E.I. Paltsev *et al.*, "Intracranial phyisology and biomechanics: Clinical data on pressure volume relationships and their interpretation", *J. Neurosurgery,* vol.57, pp. 500-510, 1982.

ACKNOWLEDGMENT

The first author would like to acknowledge Prof Sneh Anand of IIT Delhi for her suggestion and wife Lalita for helping in the preparation of this manuscript

Dr. Bhupal Singh is currently Professor and Head in the Department of Electrical and Electronics Engineering, Ajay Kumar Garg Engineering College. He is PhD (Biomedical Instrumentation and Control) from IIT Delhi, ME (Control & Instrumentation, Electrical Engineering) from MNNIT Allahabad and B.Tech. (Electrical Engineering) from IET, Lucknow. His areas of interest include: Digital Signal Processing, Intelligent Instrumentation, Non-linear Control Systems and Biomedical Instrumentation.

Dr. Sneh Anand is former Professor IIT Delhi. She is PhD (Biomedical Engineering) IITD, M. Tech (Control System) IITD and BE Electrical Engineering from Punjab Engineering College. Her areas of interest include Biomedical Instrumentation, Rehabilitation Engineering, Biomedical Transducers and Sensors, Biomechanics Technology in Reproduction Research and Controlled Drug Delivery System.

Dr. K.B. Sahay is former Professor IIT Delhi. He is PhD (Biomedical Engineering) University of Monash, Australia and B. Tech (Mechanical Engineering) IIT Kharagpur. His areas of interest include Morphological mechanics, Brain Mechanics and Population problems.