# Amplitude Equalizers - Types and Design 

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#### Abstract

Various types of amplitude equalizers (AEs) and their designs are reviewed. First the design of voltage mode AEs derived from specific OA circuits is described. Design of one of these circuits allows the realization of AEs for specified whole range and the value of variable resistance at which the flat response is obtained. However, there is a limited flexibility of choosing the shaping function $\boldsymbol{H ( s )}$. Then the block diagram approach, where one can choose $H(s)$ is presented. Next, AEs with other devices, such as FTFN, CFA, Current conveyors, etc. are derived. Then voltage mode AEs, which have virtual ground, are converted in to current mode AEs. Two additional current mode AEs are also included.


Keywords: Equalizers, Voltage mode, Current mode, Current conveyers.

## I. INTRODUCTION

THE amplitude equalizers (AEs) are used in many systems to compensate for the deviations produced in the loss-gain response. Bode [1] suggested, for realizing AE, the transfer function
$T(s)=\frac{1+x H(s)}{x+H(s)}$
where $x$ is a function of a single variable resistor $R_{v}$ and is dimensionless. For any AE, $T(s)$ should satisfy the following relations.
$T(s)=\left\{\begin{array}{c}1 / H(s) \text { for } R_{v}=R_{v 1}=0 \\ H(s) \text { for } R_{v}=R_{v 2}=\infty \\ 1 \text { for } R_{v}=R_{f}\end{array}\right.$

From eqn. (1), the following properties are noted.

1. It has a symmetry around 0 dB line and has flat response for $R_{v}=R_{f}$
2. As $x$ varies from 0 to $\infty, T(s)$ varies from $1 / H(s)$ to $H(s)$. The whole range (WR) is defined as a difference in the values of the variable resistance $R_{v}$ when $T(s)=H(s)$ and $1 / H(s)$. Thus, the WR is $\infty$.
3. The $x$ and $H(s)$ are interchangeable.
4. When $H(s)$ is replaced by $1 / H(s), T(s)$ becomes $1 / T(s)$. Hence, one has to consider the realization of either $T(s)$ or $1 / T(s)$.
5. A flat response, $T(s)=1$, is obtained when $x=1$. The corresponding value of $R_{v}$ is designated as RF.

## II. DESIGN

Analysis-based Design: Choose a suitable circuit and find its transfer function $T(s)$. Now there are following two approaches.

Approach 1: Arrange it in the form as
$T(s)=K \frac{1+x_{a} H_{a}(s)}{x_{b}+H_{b}(s)}$
Compare eqn. (3) with eqn. (1) and get the following relations.
$K=1, x=x_{a}$, for $H=H_{a}$, and $x=x_{b}$, for $H=H_{b}$,
Since there are only 3 equations, unknowns more than 3 can be suitably assumed.

Approach 2: Find three conditions from eqn. (3) as per the relations given in eqn. (2) and solve for the values of various components. Unknowns more than 3 can be suitably assumed.

We will use one or the other method in the following types of AEs.

Type I: Consider the circuit shown in Fig. 1. Analysis gives

$$
\begin{gather*}
T_{1}=\frac{V_{o}}{V_{i}}=\frac{Z\left(R_{a}+R_{b}\right)+R_{a} R_{b}+Z R}{Z\left(R_{a}+R_{b}\right)+R_{a} R_{b}+Z R+R_{a} R} \\
=\frac{Z\left(R_{a}+R_{b}\right)+R_{a} R_{b}+Z R}{\left(Z+R_{a}\right) R+Z\left(R_{a}+R_{b}\right)+R_{a} R_{b}} \\
{\left[\frac{Z\left(R_{a}+R_{b}\right)+R_{a} R_{b}}{\left(Z+R_{a}\right) R a}\right] \frac{Z}{\left[\frac{R}{R_{a}}+\frac{R}{R_{a}}\right)\left(\frac{Z\left(1+\frac{R_{b}}{R_{a}}\right)+R_{b}}{\left(Z+R_{a}\right)}\right]}} \tag{4}
\end{gather*}
$$

Let

$$
\begin{align*}
& \frac{Z\left(R_{a}+R_{b}\right)+R_{a} R_{a}}{\left(Z+R_{a}\right) R_{a}}=1,  \tag{5}\\
& \Rightarrow Z R_{b}=R_{a}\left(R_{a}-R_{b}\right)
\end{align*}
$$



Figure 1. Passive circuit.
Equation (5) demands $R_{a}=R_{b}=0$, which is not admissible. To overcome this difficulty, we take $R=R_{v}-R_{o}$.
Now eqn. (4) becomes

$$
\begin{align*}
& T_{2}(s)=\left[\frac{Z\left(R_{a}+R_{b}-R_{o}\right)+R_{a} R_{b}}{\left(Z+R_{a}\right) R_{a}}\right] \\
& {\left[1+\left(\frac{R_{v}}{R_{a}}\right)\left(\frac{Z}{Z\left\{1+\frac{R_{b}-R_{o}}{R_{o}}\right\}+R_{b}}\right)\right]}  \tag{6}\\
& {\left[\frac{R_{v}}{R_{a}}+\frac{Z\left\{1+\frac{R_{b}-R_{o}}{R_{o}}\right\}+R_{b}-R_{o}}{\left(Z+R_{a}\right)}\right]}
\end{align*}
$$

Let

$$
\begin{aligned}
& \frac{Z\left(R_{a}+R_{b}-R_{o}\right)+R_{a} R_{b}}{\left(Z+R_{a}\right) R_{a}}=1 \\
& x_{2}=\frac{R_{v}}{R_{a}} \\
& H_{2}(s)=\frac{Z}{Z\left(1+\frac{R_{b}-R_{o}}{R_{a}}\right)+R_{b}} \\
& =\frac{Z\left(1+\frac{R_{b}-R_{o}}{R_{a}}\right)+\left(R_{b}-R_{o}\right)}{Z+R_{a}}
\end{aligned}
$$

Now eqn. (7) demands

$$
Z\left(R_{b}-R_{o}\right)=R_{a}\left(R_{a}-R_{b}\right) .
$$

This can be satisfied only if

$$
R_{a}=R_{b}=R_{o}
$$

Then eqn. (6) reduces to

$$
\begin{equation*}
x_{2}=\frac{R_{v}}{R_{o}} \tag{10}
\end{equation*}
$$

and eqn. (9) to
$H_{2}=\frac{Z}{Z+R_{o}}$
Finally,

$$
\begin{equation*}
T_{2}(s)=\frac{1+x_{2} H_{2}}{x_{2}+H_{2}} \tag{11}
\end{equation*}
$$

Thus, the circuit reduces to that shown in Figure 2 which is the same as proposed by Saraga and Zyoute [2]. The fixed negative resistance $-R_{0}$ can be simulated using a negative resistance converter. Since the negative resistance is a floating one, it will require too many components to simulate it. The circuit requires an additional buffer at the output to avoid loading. The WR of this AE is $\infty$.


Figure 2. AE 1 (Saraga and Zyoute [2]).
Type II: Consider the circuit shown in Fig. 3(a). Analysis gives
$T_{3}(s)=\frac{R-\frac{R_{b}}{R_{a}} Z}{R+Z}$
Let
$R_{a}=R_{b}=R_{o}$
Then

$$
\begin{align*}
& T_{3}(s)=\frac{R-Z}{R+Z} \\
& \quad=\frac{2 R_{o}(R-Z)+\left(R_{o}{ }^{2}+R Z\right)-\left(R_{o}{ }^{2}+R Z\right)}{2 R_{o}(R-Z)+\left(R_{o}{ }^{2}+R Z\right)-\left(R_{o}{ }^{2}+R Z\right)} \\
& =\frac{1+\left(\frac{R+R_{o}}{R-R_{o}}\right)\left(\frac{R_{o}-Z}{R_{o}+Z}\right)}{\left(\frac{R+R_{o}}{R-R_{o}}\right)+\left(\frac{R_{o}-Z}{R_{o}+Z}\right)}  \tag{14}\\
& =\frac{1+x_{3} H_{3}}{x_{3}+H_{3}}
\end{align*}
$$


(a)

(b)

Figure 3. (a) Active circuit and (b) AE 2 (Brglez[3]).
where

$$
\begin{equation*}
x_{3}=\frac{R+R_{o}}{R-R_{o}} \tag{15}
\end{equation*}
$$

and

$$
\begin{equation*}
H_{3}=\frac{R_{o}-Z}{R_{o}+Z} \tag{16}
\end{equation*}
$$

Thus, we note that any first order all-pass transfer function (when $\mathrm{Z}=1 / s C$ ) can be converted into an AE function by introducing $R_{o}$.

From eqn. (16), note that for $x_{3}$ to vary from 0 to $\infty$ so that $T_{3}(s)$ varies from $1 / H_{3}(s)$ to $H_{3}(s), R$ should vary from $-R_{0}$ to $+R_{0}$. Thus WR is $2 R_{0}$. Since, a practical variable resistor cannot have negative value, we take
$R=R_{v}-R_{o}$
In view of eqn. (17a) and (14) becomes

$$
\begin{equation*}
T_{3}(s)=\frac{1+\left(\frac{R_{v}}{R_{v}-2 R_{0}}\right) H_{3}(s)}{\frac{R_{v}}{R_{v}-2 R_{0}}+H_{3}(s)} \tag{17b}
\end{equation*}
$$

The $T_{3}(s)$ now varies from $1 / H_{3}(s)$ to $H_{3}(s)$ when $R_{v}$ varies from 0 to $2 R_{0}$. Thus, the WR remains the same $2 R_{0}$. A flat response is obtained when $R_{v}=R_{0}$. Thus the circuit shown in Fig. 3(a) leads to the circuit shown in Fig. 3(b) which is the same as given by Brglez [3]. It may be noted that the Brglez Circuit uses $Z$ as a
series combination of the fixed resistance $R_{o}$ and impedance $Z$. This splitting of $Z$ may not be required.

NIC is simulated using a CFA as shown in Figu. 3(b) [4]. To eliminate the NIC in the circuit of Fig. 3(b), Brglez [5] used a switch as shown in Fig. 4. The circuit provides the positive $R_{v}$ range when the switch is connected to A terminal and the negative $R_{v}$ range when connected to B terminal. One can see that the toggling of the switch provides basically two inverse active networks [6].


Figure 4. AE 2 (Brglez AE with a switch).
Type III: Consider the circuit [7] shown in Fig. 5.
The analysis of the circuit gives
$T_{4}(s)=\left[\frac{Z+R_{b}}{Z+R_{c}+R_{e}}\right] \frac{\left[1+\left(\frac{R_{v}}{R_{e}}\right)\left(\frac{Z+R_{e}-\frac{R_{b} R_{c}}{R_{a}}}{Z+R_{b}}\right)\right]}{\left[\frac{R_{v}}{R_{e}}+\frac{Z}{Z+R_{c}+R_{e}}\right]}$
Design 1: Let

$$
\begin{equation*}
\frac{Z+R_{b}}{Z+R_{c} R_{e}}=1 \tag{18b}
\end{equation*}
$$

$$
\begin{align*}
\rightarrow R_{b} & =R_{c}+R_{e}  \tag{19}\\
x_{4} & =\frac{R_{v}}{R_{e}} \tag{20}
\end{align*}
$$

and

$$
\begin{align*}
H_{4}(s)=\frac{Z}{Z+R_{b}} & =\left(\frac{Z+R_{e}-\frac{R_{b} R_{c}}{R_{a}}}{Z+R_{b}}\right)  \tag{21}\\
\rightarrow R_{b} & =\frac{R_{a} R_{e}}{R_{c}} . \tag{22}
\end{align*}
$$

Equating the two values of $R_{b}$ from eqn. (19) and eqn. (22), we get
$R_{c}^{2}+R_{e} R_{c}-R_{a} R_{e}=0$.

Solving for $R_{c}$, we get

$$
\begin{align*}
& R_{c}=\frac{-R_{e} \pm \sqrt{R_{e}^{2}+4 R_{a} R_{e}}}{2}  \tag{24}\\
& T_{4}(s)=\frac{1+\left(\frac{R_{v}}{R_{e}}\right)\left(\frac{Z}{Z+R_{b}}\right)}{\left(\frac{R_{v}}{R_{e}}\right)+\left(\frac{Z}{Z+R_{b}}\right)}
\end{align*}
$$

Equations (23) and (24) are the design relations for AE of Figure $5(\mathrm{a})$, after choosing suitable


Figure 5. AE 3.
Value for $R_{a}$. Thus, there are many possible AEs. Two of them are given below.
(a) AE 3(a) Let $R_{a}=2 R_{e}$. Then $R_{\mathrm{c}}=R_{e}, R_{b=} 2 R_{e}$. This circuit is the same as given by Zyoute [7] when $R_{e}=R_{o}$.
(b) AE 3(b): Let $R_{a}=(15 / 4) R_{e}$. Then $R_{c}=(3 / 2) R_{e}, R_{b}=(5 / 2) R_{e}$.

Design 2: Equation (17) can be rearranged as

$$
\begin{equation*}
T_{5}(s)=K \frac{1+x_{a} H_{5}(s)}{x_{b}+H_{5}(s)} \tag{25}
\end{equation*}
$$

where

$$
\begin{align*}
& K= \frac{1+\frac{R_{v} R_{a} R_{b} R_{e}-R_{b}^{2} R_{c} R_{v}-R_{v} R_{a} R_{e} Z}{R_{a} R_{b} R_{e}\left(Z+R_{b}\right)}}{1-\frac{R_{v}}{R_{b}}}  \tag{26}\\
& x_{a}=\frac{\frac{R_{v}}{R_{b}}\left(1+\frac{R_{b}}{R_{e}}\right)}{\left(1-\frac{R_{v}}{R_{b}}\right)\left(Z+\frac{R_{b}-\frac{R_{b} R_{c} R_{v}}{R_{a} R_{e}}+R_{v}}{1-\frac{R_{v}}{R_{b}}}\right)}  \tag{27}\\
& x_{b}=\frac{\left(\frac{R_{v}}{R_{b}}\right]\left[1+\left(\frac{R_{b}}{R_{e}}\right)\left(\frac{Z+R_{c}}{Z+R_{b}}\right)\right]}{1-\frac{R_{v}}{R_{b}}}  \tag{28}\\
& \text { nd }
\end{align*}
$$

$$
\begin{equation*}
H_{5}(s)=\frac{Z}{Z+R_{b}} \tag{29}
\end{equation*}
$$

Let

$$
\begin{equation*}
x_{5}=x_{a}=x_{b}, \tag{3}
\end{equation*}
$$

i.e.,

$$
\begin{align*}
x_{5} & =\frac{\frac{R_{v}}{R_{b}}\left(1+\frac{R_{b}}{R_{e}}\right)}{\left(1-\frac{R_{v}}{R_{b}}\right)\left(Z+\frac{R_{b}-\frac{R_{b} R_{c} R_{v}}{R_{a} R_{e}}+R_{v}}{1-\frac{R_{v}}{R_{b}}}\right)} \\
& =\frac{\frac{R_{v}}{R_{b}}\left[1+\left(\frac{R_{b}}{R_{e}}\right)\left(\frac{Z+R_{c}}{Z+R_{b}}\right)\right]}{1-\frac{R_{v}}{R_{b}}} \tag{31}
\end{align*}
$$

For $x_{5}$ to be independent of $Z$, eqn. (30) requires $R_{b}=\frac{2 R_{a} R_{e}}{R_{c}}$
and

$$
\begin{equation*}
R_{b}=R_{c}=R_{o} \tag{33}
\end{equation*}
$$

Equating the two values of $R_{b}$ from eqns. (32) and (33), we get
$R_{c}=\sqrt{2 R_{a} R_{e}}$.
Under these conditions eqn. (25) gives
$K=1$.
Thus from eqn. (30)

$$
\begin{equation*}
x_{5}=\frac{\frac{R_{v}}{R_{b}}\left\lfloor 1+\left(\frac{R_{b}}{R_{e}}\right)\right\rfloor}{1-\frac{R_{v}}{R_{b}}}=\frac{R_{b}+R_{e}}{\frac{R_{b} R_{e}}{R_{v}}-R_{e}} . \tag{35}
\end{equation*}
$$

Now eqn. (25) becomes

$$
\begin{equation*}
T_{5}(s)=\frac{1+x_{5} H 5(s)}{x_{5}+H_{5}(s)} \tag{36}
\end{equation*}
$$

where $H_{5}$ and $x_{5}$ are given by eqn. (29) and (35), respectively. From eqn. (35), note that $T_{s}(s)$ varies from $1 / H_{s}(s)$ to $H_{s}(s)$ when $R_{v}$ varies from 0 to $R_{o}$. Thus, the WR becomes $R_{o}$.

The design steps are the following.
(i) Choose $R_{a}$ and $R_{e}$.
(ii) Find $R_{c}$ from eqn. (34)
(iii) Find $R_{b}$ from eqn. (32).
(a) AE 3(c): Let $R_{a}=R_{o} / 2, R_{e}=R_{o}$ then $R_{c}=R_{b}=R_{o}$. The circuit becomes the same as depicted by Talkhan et al. [8].
(b) AE 3(d): Let $R_{a}=2 R_{e}, R_{e}=R_{o}$, then we get $R_{b}=R_{c}=2 R_{e}$.

Design 3: Equation (17) can be rearranged as

$$
\begin{equation*}
T_{6}(s)=\frac{Z+\frac{R_{a} R_{b} R_{e}+R_{v}\left(R_{a} R_{e}-R_{b} R_{c}\right)}{R_{a}\left(R_{v}+R_{e}\right)}}{Z+\frac{R_{v}\left(R_{e}+R_{c}\right)}{\left(R_{v}+R_{e}\right)}} \tag{37}
\end{equation*}
$$

From eqn. (37),

$$
\begin{gather*}
T_{6}(s)_{R_{v}=0}=\frac{Z+R_{b}}{Z}=H(s)  \tag{38}\\
T_{6}(s)_{R_{v}=R_{r}} \\
=\frac{Z+\frac{R_{a} R_{b} R_{e}+R_{r}\left(R_{a} R_{e}-R_{b} R_{c}\right)}{R_{a}\left(R_{r}+R_{e}\right)}}{Z+\frac{R_{r}\left(R_{e}+R_{c}\right)}{\left(R_{r}+R_{e}\right)}}  \tag{39}\\
=\frac{1}{H(s)}=\frac{Z}{Z+R_{b}}
\end{gather*}
$$

This is satisfied when
$\frac{R_{a} R_{b} R_{e}+R_{r}\left(R_{a} R_{e}-R_{b} R_{c}\right)}{R_{a}\left(R_{r}+R_{e}\right)}=0$
$\Rightarrow \quad R_{b}=\frac{R_{r} R_{e} R_{a}}{R_{r} R_{c}-R_{a} R_{e}}$
and
$R_{b}=\frac{R_{r}\left(R_{e}+R_{c}\right)}{\left(R_{r}+R_{e}\right)}$
Equating two values of $R_{b}$ from eqns. (45) and (46), we get
$R_{a}=\frac{R_{c} R_{r}\left(R_{e}+R_{c}\right)}{R_{e}\left(R_{r}+2 R_{e}+R_{c}\right)}$
Condition for flat response, from eqn. (36), is

$$
\begin{align*}
& T_{6}(s)_{R_{v}=R_{f}} \\
& =\frac{Z+\frac{R_{a} R_{b} R_{e}+R_{f}\left(R_{a} R_{e}-R_{b} R_{c}\right)}{R_{a}\left(R_{f}+R_{e}\right)}}{Z+\frac{f\left(R_{e}+R_{c}\right)}{\left(R_{f}+R_{e}\right)}}=1  \tag{44}\\
& \Rightarrow R_{e}=\frac{R_{f} R_{r}}{R_{r}-2 R_{f}} \tag{45}
\end{align*}
$$

For $R_{e}$ to be non-negative real,
$R_{r} \geq 2 R_{f}$

Thus
$\left.T_{6}(s)\right|_{R_{r}=2 R_{f}}=\frac{Z+R_{b}+R_{f}}{Z+R_{f}}>1$
and eqn. (43) is not satisfied. Therefore

$$
\begin{equation*}
R_{r}>2 R_{f} \tag{48}
\end{equation*}
$$

Equations (42), (43) and (45) are the design equations with restrictions given by eqn. (48).

Let

$$
\begin{equation*}
R_{b}=R_{o} \tag{49}
\end{equation*}
$$

Then from eqn. (42), we get

$$
\begin{equation*}
R_{c}=R_{o}+\left(\frac{R_{o}}{R_{r}}-1\right) R_{e} \tag{50}
\end{equation*}
$$

Design procedure, when $R_{r}$ and $R_{f}$ are specified, is as follows [9].

Find $R_{e}$ from eqn. (45), $R_{b}$ from eqn. (49), $R_{c}$ from eqn. (50) and $R_{a}$ from eqn. (43).

Following the above procedure, $\mathrm{AE}-3(e)-\mathrm{AE} 3(i)$ are designed for five different sets of values of $R_{r}$ and $R_{f}$ and results are given in Table 1. Value of $x$ is obtained from $x_{6}$ given in eqn. (35).

AE 3(e) is the same as that of Zyoute [7]. However, he has chosen the value of $R_{e}$, and therefore, $R_{f}$ gets fixed as per eqn. (45). AEs $3(f)$ and $3(g)$ were considered in [8] where different values of $R_{e}$ gives different $R_{f}$ as per eqn. (45), but $R_{r}$ remains the same. In [7] and [8], $R_{e}$ or $R_{a}$ was chosen instead of finding it from eqn. (45) or eqn. (43).

It may be noted that the design is applicable for both the fan and bump equalizers and also for the inverse networks [6].

## More complex AEs

Type IV: AE-4 proposed by Nowrouzian and Fuller [10] is, after correction, shown in Fig. 6. Analysis of the circuit yields

$$
\begin{equation*}
H_{7}(s)=\frac{R_{0}+Z}{R_{0}-Z} \tag{51}
\end{equation*}
$$

It requires, after including one missing inverter in their circuit, 5 OAs and 14 resistors. In the circuit of Fig. 6(b), one cannot isolate the block $H_{7}(s)$.

TABLE 1 -- DESIGN OF EQUALIZERS FOR FIVE SETS OF $R_{r}$ AND $R_{f}$, WITH $R_{b}=R_{o}$

|  | $R_{r}$ | $R_{f}$ | $R_{e}$ | $R_{c}$ | $R_{a}$ | $x$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: |
| AE 3(e) | $\infty$ | $\frac{1}{2} R_{0}$ | $\frac{1}{2} R_{0}$ | $\frac{1}{2} R_{0}$ | $R_{o}$ | $\frac{2 R_{v}}{R_{0}}$ |
| AE 3(f) | $R_{o}$ | $\frac{1}{3} R_{0}$ | $R_{o}$ | $R_{o}$ | $\frac{1}{2} R_{0}$ | $\frac{2 R_{v}}{R_{0}-R_{v}}$ |
| AE 3(g) | $R_{o}$ | $\frac{1}{4} R_{0}$ | $\frac{1}{2} R_{0}$ | $R_{o}$ | $R_{o}$ | $\frac{3 R_{v}}{R_{0}-R_{v}}$ |
| AE 3(h) | $\frac{1}{2} R_{0}$ | $\frac{1}{8} R_{0}$ | $\frac{1}{4} R_{0}$ | $\frac{5}{4} R_{0}$ | $\frac{5}{3} R_{0}$ | $\frac{6 R_{v}}{R_{0}-2 R_{v}}$ |
| AE 3(i) | $\frac{1}{4} R_{0}$ | $\frac{1}{10} R_{0}$ | $\frac{1}{2} R_{0}$ | $\frac{5}{2} R_{0}$ | $R_{0}$ | $\frac{6 R_{v}}{R_{0}-4 R_{v}}$ |



Figure 6. AE 4.


Figure 7. AE 5.


Figure 8. AE 6.

TABLE 2 -- A COMPARISON OF VARIOUS AEs

| AEs | No |  | WR | $H(s)$ |
| :---: | :---: | :---: | :---: | :---: |
|  | OAs | Resistors |  |  |
| Saraga and Zyoute <br> AE [2] | 01 | 06 | $\infty$ |  |
| Brglex AE [3] | 02 | 08 | $2 R_{0}$ | $R_{0} /\left(Z+R_{0}\right)$ |
| Zyoute AE [7] | 01 | 05 | $\infty$ | $Z /\left(Z+2 R_{0}\right)$ |
| Talkhan AE [8] | 01 | 05 | $\mathrm{R}_{0}$ | $Z /\left(Z+R_{0}\right)$ |
| Nowrouzian and <br> Fuller AE [10] | 05 | 14 | $\infty$ | $\frac{R_{o}+Z}{R_{o}-Z}$ |
| Nowrouzian et al. <br> AE [11] | 03 | 11 | $\infty$ | $\frac{2 Z}{2 Z+R_{o}}$ |
| Rathore and Khot <br> AE [13] | 04 | 12 | $\infty$ | $\frac{R_{o}+Z}{R_{o}-Z}$ |

Type V: AE 5, with the similar performance as AE 4, but with reduced number of components, is shown in Fig. 7.

Type VI: AE 6, proposed by Nowrouzian et al. [11], is shown in Fig. 8. The analysis of the circuit leads to

$$
\begin{equation*}
T_{7}(s)=-\frac{\frac{R_{v}}{R_{0}}+\frac{2 Z}{2 Z+R_{0}}}{1+\left(\frac{R_{v}}{R_{0}}\right) \frac{2 Z}{2 Z+R_{0}}}=-\frac{x_{7}+H_{7}(s)}{1+x_{7} H_{7}(s)} \tag{52}
\end{equation*}
$$

where

$$
\begin{equation*}
x_{7}=\frac{R_{v}}{R_{0}} \tag{53}
\end{equation*}
$$

and

$$
\begin{equation*}
H_{7}(s)=\frac{2 Z}{2 Z+R_{0}} \tag{54}
\end{equation*}
$$

## Block diagram approach

In all the above types of AEs, there is very limited flexibility of choosing $H(s)$ by choosing $\mathrm{Z}(\mathrm{s})$. Now we will introduce AEs which can have any desired $H(s)$ [12]. The block diagram of the function

$$
\begin{equation*}
T_{7}(s)=\frac{\frac{R_{v}}{R_{o}}+H_{7}(s)}{1+\frac{R_{v}}{R_{o}} H_{7}(s)} \tag{55}
\end{equation*}
$$

is given in Fig. 9. Note that we have chosen all summers with inverting type so that they can be realized by inverting type OA configuration. Since there are four summers, we will require minimum 4 OAs.

Type VII: Based on the block diagram, one possible realization of is shown in Fig. 10(a). In this circuit

$$
\begin{equation*}
x_{7}=\frac{R_{v}}{R_{o}} \tag{56}
\end{equation*}
$$

One may choose

$$
\begin{equation*}
H_{7}(s)=\frac{R_{o}+Z}{R_{o}-Z} \tag{57}
\end{equation*}
$$

A circuit for realizing $H_{7}(s)$ is shown in Fig.10(b) with noninverting terminals of all the OAs grounded. If this $H_{7}(s)$ block of Fig. 10(c) is inserted into the main circuit in Fig. 10(a), one inverter and one summer can be eliminated. Thus the circuit can be reduced to that of [10].

Type VIII: One may choose
$H_{8}(s)=\frac{1}{H_{7}}=\frac{R_{O}-Z}{R_{o}+Z}$.
Then
$T_{8}(s)=\frac{1}{T_{7}(s)}=\frac{1+\frac{R_{v}}{R_{o}} H_{8}(s)}{\frac{R_{v}}{R_{o}}+H_{8}(s)}$
The realization is obtained by inverse transform [6] on Fig. $10(b)$ and shown in Fig. 10(c).


Figure 9. Block diagram of $\mathrm{T}_{7}(\mathrm{~s})$.


Figure 10. Active RC realizations of $(a) T_{7}(s),(b) H_{7}(s)$ and (c) $1 / H_{7}(\mathrm{~s})$.

## AES with other active devices

## Type IX

Circuit VII: The OA based AE shown in Fig. 5 (reproduced in Fig. 11(a) can be transformed into circuits employing other active devices. Two such circuits employing active devices having terminal characteristics $V_{x}=V_{y}, I_{z}=I_{x}$ and $I_{y}=0$ are derived and shown in Fig. 11(b) and $11(c)$.

From figures $11(a)$, (b) and (c), we can write, respectively,

$$
\begin{align*}
& I=I_{1}+I_{2}=\left(\frac{V_{x}}{R_{b}}-\frac{V_{o}}{R_{b}}\right)+I_{2}  \tag{60}\\
& I=I_{1}+I_{2}=\left(I_{x}+I_{z}\right)+I_{2} \\
& \quad=2 I_{z}+I_{2}=2\left(\frac{V_{x}}{R}-\frac{V_{o}}{R}\right)+I_{2}  \tag{61}\\
& I=I_{1}+I_{x}+I_{2}=I_{1}+I_{z}+I_{2} \\
&  \tag{62}\\
& =\left(\frac{V_{x}}{R_{1}}-\frac{V_{o}}{R_{2}}\right)+I_{2}
\end{align*}
$$

For the circuits shown in Figs. 11(a) and (b) to be equivalent to that shown in Fig. 11(a), eqns. (60) and (61) require
$R=2 R_{b}$
and from eqns. (60) and (62),
$R_{1}=R_{2}=R_{b}$

Although the circuit of Fig. 13(c) requires one more resistance than that shown in Fig. 13(b), the total resistance is the same.

(a)

(c)

Figure 11. (a) OA-based AE, (b) and (c) Two AEs derived from (a).


Figure 12 (a) VM AE obtained from the AE of Fig. 12 and (b) Some active devices.

Type X: Another type of AEs derived from OA-based circuit of Fig. 7 is shown in Fig. 12(a) [14]. Some of the active devices are shown in Fig. 12(b). In case of CFA an admittance $Y$ is added at terminal D , which is a part of the feedback admittance connected across AC. The terminal characteristics of CCII, FTFN and CFA, respectively, are

$$
\left[\begin{array}{l}
V_{x}  \tag{65}\\
I_{z} \\
I_{y}
\end{array}\right]=\left[\begin{array}{lll}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
V_{y} \\
I_{x} \\
V_{z}
\end{array}\right], \quad\left[\begin{array}{c}
V_{x} \\
I_{z} \\
I_{y} \\
V_{w}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1
\end{array}\right]\left[\begin{array}{c}
V_{y} \\
I_{x} \\
I_{w} \\
V_{z}
\end{array}\right], \quad\left[\begin{array}{c}
V_{x} \\
I_{z} \\
I_{y} \\
I_{x}
\end{array}\right]=\left[\begin{array}{cccc}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{array}\right]\left[\begin{array}{c}
V_{y} \\
I_{w} \\
V_{w} \\
V_{z}
\end{array}\right]
$$

## Current mode AEs using VM-CM transformation

Type XI: Using VM-to-CM transformation method [13], one can convert the OA based VM AE of Fig. 7 into CM AE as show in Fig. 13(a) with different active devices as shown in Fig. 13(b) [14]. The terminal characteristic of OA is $V_{x}=V_{y} I_{x}=I_{y}=0$. The characteristics of other devices are given in eqn. (65).

Type XII
Now, consider the circuit using $\operatorname{CDBA}\left(V_{x}=0, V_{y}=0, I_{z}=\right.$ $I_{y}-I_{x}, V_{w}=V_{z}$ ) as shown in Fig. 14(a). The analysis of the circuit leads to

$$
\begin{equation*}
T(s)=\frac{I_{o}}{I_{i}}=\frac{\frac{R_{v}}{Z}-1}{\frac{R_{v}}{Z}+1} \tag{66}
\end{equation*}
$$

This $T(s)$ is in similar form as that given in eqn. (17) when $R_{a}=R_{b}$. Following the similar technique used for realizing

Brglez's AE, we get
$T(s)=\frac{1+\left(\frac{R_{p}}{R_{p}-2 R_{0}}\right)\left(\frac{R_{0}-Z}{R_{0}+Z}\right)}{\frac{R_{p}}{R_{p}-2 R_{0}}+\frac{R_{0}-Z}{R_{0}+Z}}$
where $R_{v}=R_{p}-R_{o}$.
Comparing eqn. (67) with eqn. (1), we get

$$
\begin{equation*}
x=\frac{R_{p}}{R_{p}-2 R_{0}} \tag{68}
\end{equation*}
$$

and

$$
\begin{equation*}
H(s)=\frac{R_{0}-Z}{R_{0}+Z} \tag{69}
\end{equation*}
$$



Figure 13 (a). CM AEs derived from VM AE of Figure 7, (b) Some active devices.


Figure 14. CM AEs employing $(a)$ one CDBA, $(b)$ one CDBA and one NIC.

The complete CM AE is shown in Fig. 14(b). The negative resistance $R_{o}$ is simulated as shown in the figure. The WR of the AE is $2 R_{o}^{o}$.

## III. CONCLUSION

In this review paper, various types of amplitude equalizers (AEs) and their designs have been presented. First, the design of voltage mode AEs derived from specific OA circuits is described. Design of one of these circuits allows the realization of AEs for specified whole range (WR) and the value of variable resistance (RF) at which the flat response is obtained. However, there is a limited flexibility of choosing the shaping function $H(s)$. The block diagram approach, where one can choose $H(s)$, has been given. Next, AEs with other devices, such as FTFN, CFA, Current conveyors, etc. are presented. Voltage mode AEs, which have virtual ground, have been converted in to current mode AEs. Two additional current mode AEs are also presented.

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